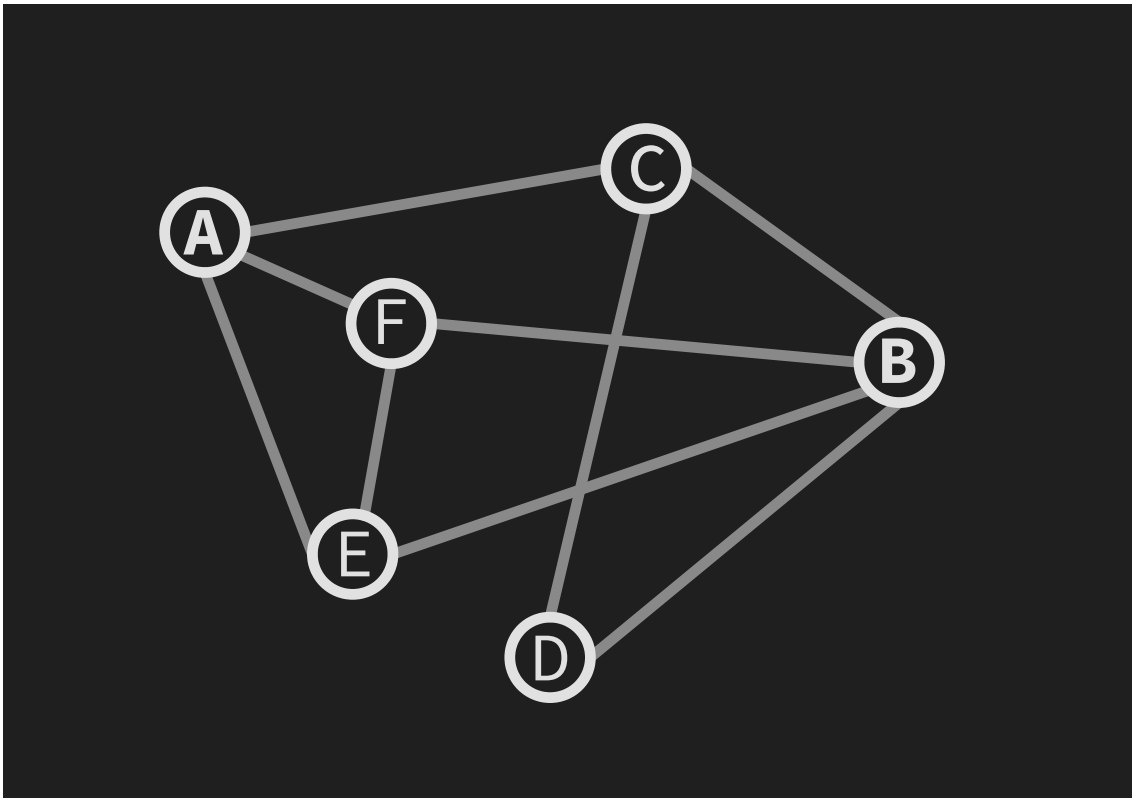
the Matching Problem

Matching Problems

What is matching?

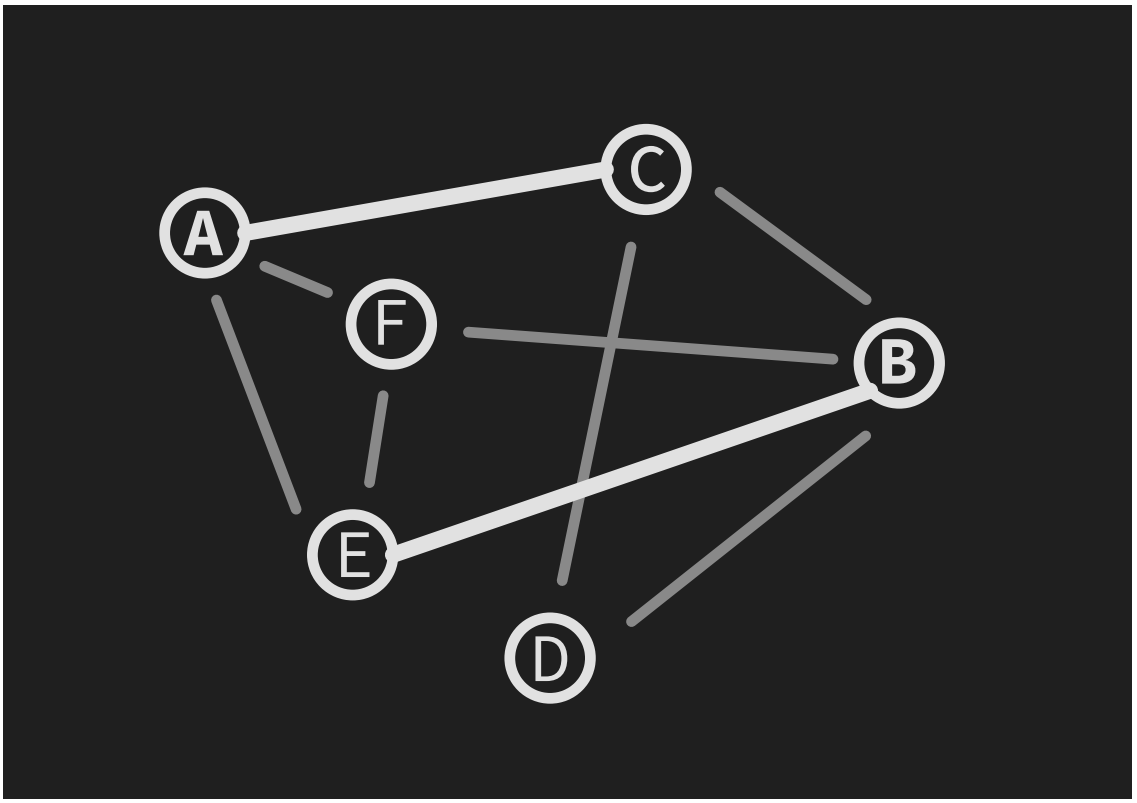
In computer science **matching** is a problem on graphs.
We want to know: can we form (exclusive) pairs of nodes in this graph so that all nodes are part of a pair? Such an assortment of pairs is called a **matching**

example 1



Look at the matching in the following example 2. What can you observe?

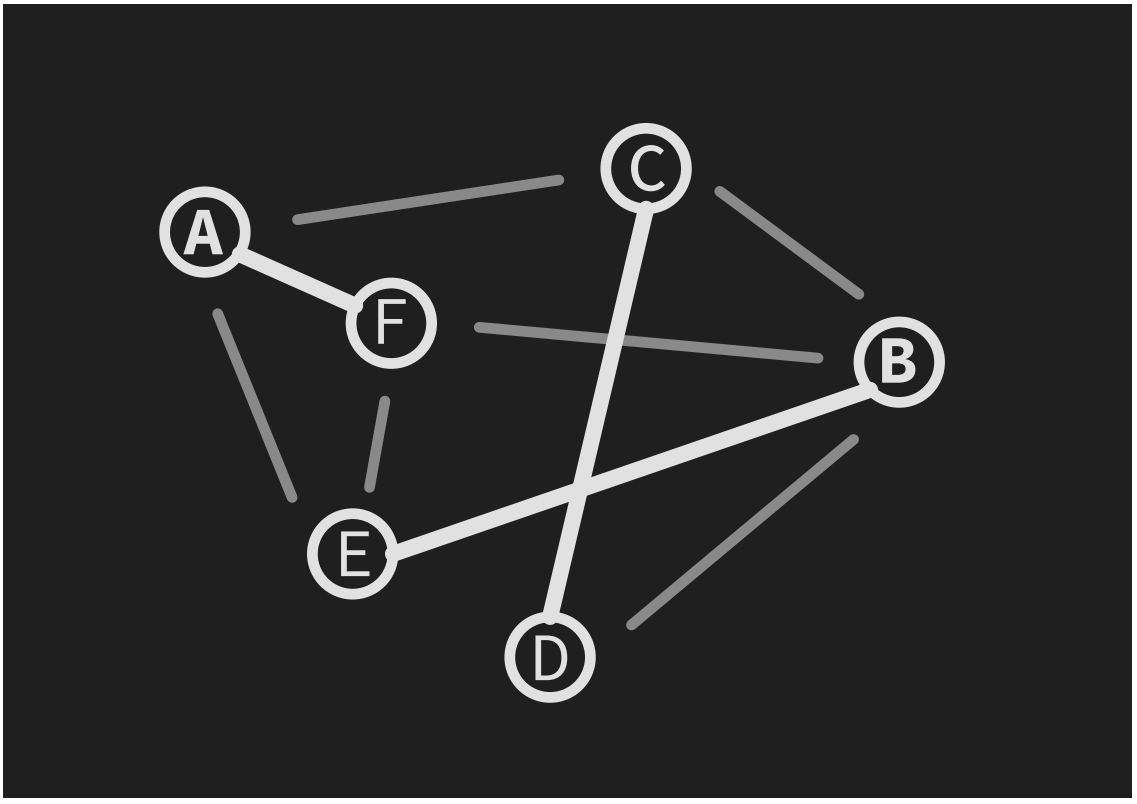
example 2



we can observe 2 matches in this matching: (A,C) and (B,E)

Is it possible to have more matchings? Indeed there is:

example 3



We can count 3 matches:
(A,F) and (B,E) and (C,D)

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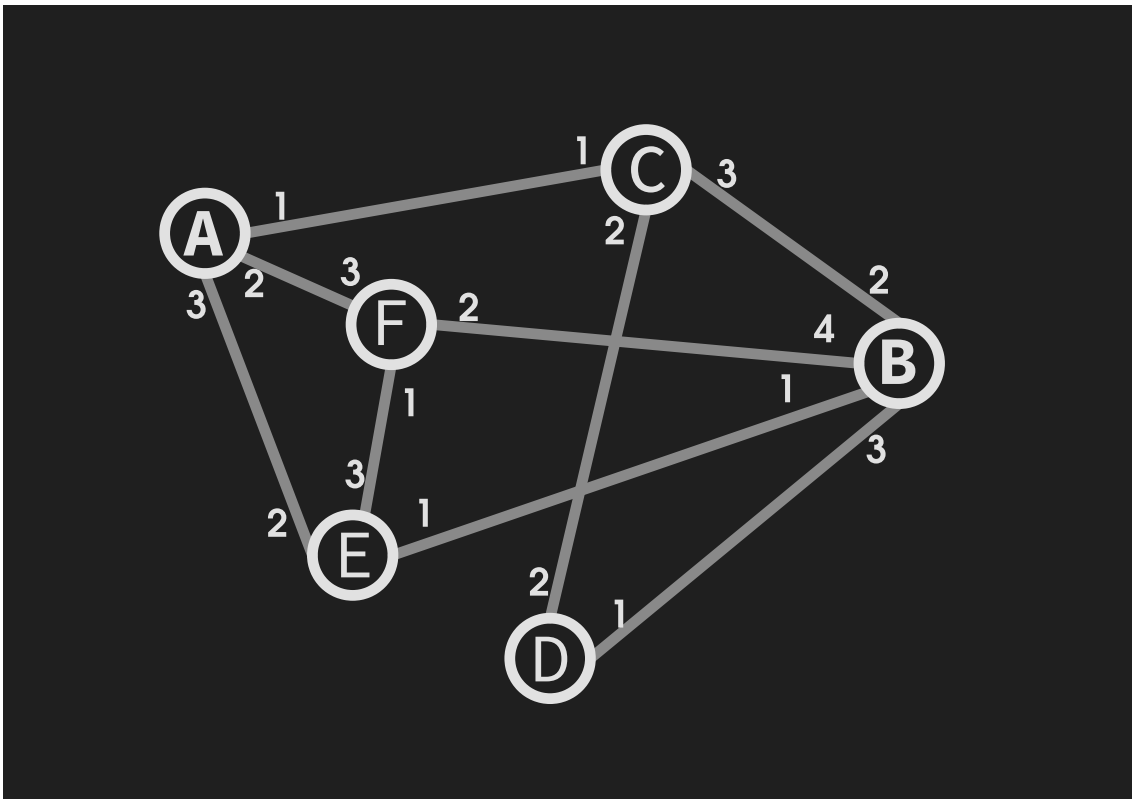
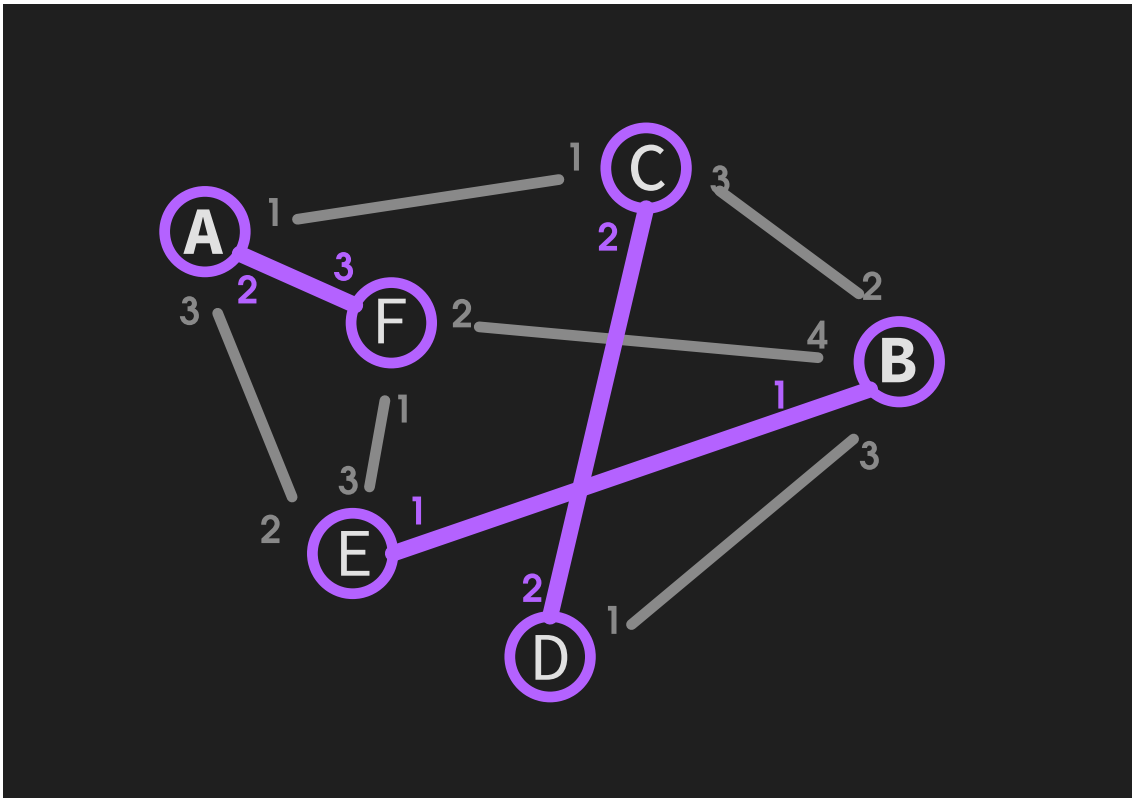
preferences



So far so good. Now let's increase the complexity by adding **preferences**.
We can also express preferences similarly to an adjancency list:
A : (C, F, E)
B : (E, C, D, F)
C : (A, D, B)
D : (B, C)
E : (B, A, F)
F : (E, B, A)

Let's return to example 3 that found a matching of nodes, and observe how the preferences impact the matching.



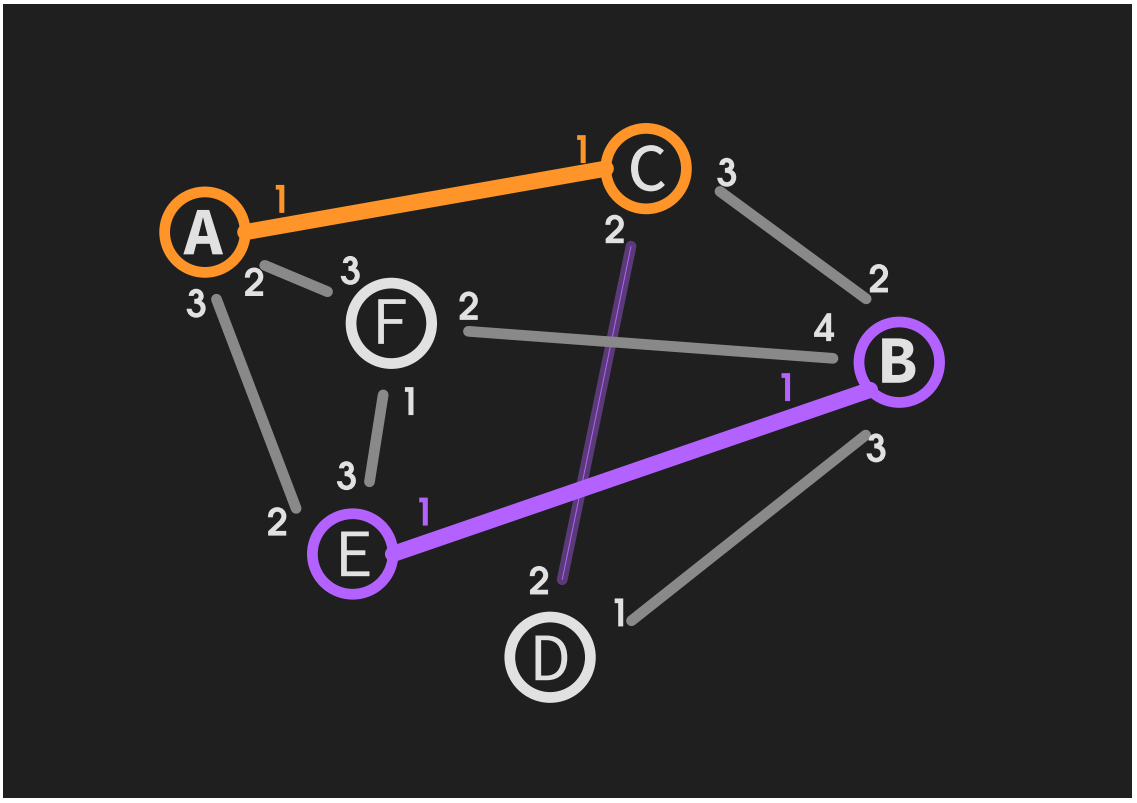
Is this the optimal solution?

unstable pairs

No! In this graph we have something called an **unstable pair**: (C,D)
Both of these nodes have a higher preference for nodes other than the one they are currently coupled with.
The complement to the unstable pair is the ...

breaking pair

A **breaking pair** comes together if a unstable pair can actualize their higher preference for other nodes.
In our example (A,C) is one such **breaking pair** For these nodes our "optimal" solution kept both of them in a pairing that was less desireable than the (A,C) pairing.



In this situation not all nodes are matched, but an optimal solution (in which a maximum amount of nodes is matched with a node with the highest possible preference) is not possible because the existence of breaking pairs.

stable matching

In the light of this, the problem definition shifts to the **stable matching problem**. Here optimality is not the goal, but **stability**. A stable matching is one, where no breaking pair "breaks up" our match.

exercise 1

Identify unstable pairs with the examples from the website here:
https://uw-cse442-wi20.github.io/FP-cs-algorithm/

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the Gale-Shapely Algorithm

In 1962, **David Gale** and **Lloyd Shapley** proved that, for any equal number of men and women, it is always possible to make all marriages stable. Let's assume we have 4 unengaged men and women:

The **Gale–Shapley algorithm** involves a number of *"rounds"*:

```
- Round 1:
    Every suitor (single man) asks his first choice.
    If the first choice is single, she always says yes.
    They are now engaged.
```

```
- Round 2
    Every suitor (single) asks his second choice
    If the second choice is single, she always says yes.
        They engage.
    else...
        if the suitor is better than her current fiancé,
            she breaks the engagement & says yes to the suitor

- Round x
    Every single man asks his x-th choice ...

...Repeat as long as there are still single men...
```

excercise 2

Execute the Gale-Shapely Algorithm on the examples on this website
https://uw-cse442-wi20.github.io/FP-cs-algorithm/

af?ect! method

What affects did you have in the face of this algorithm?

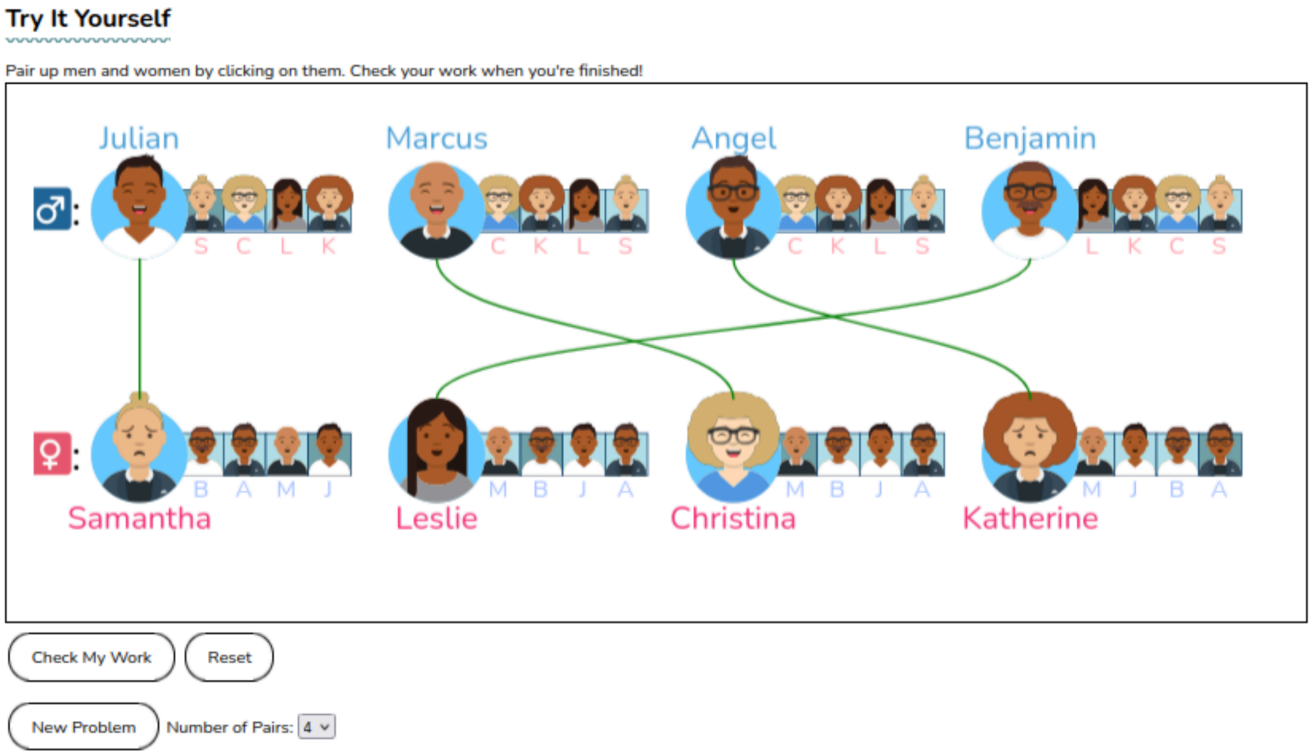
what are social assumptions in the algorithm?

algorithmic benchmarks

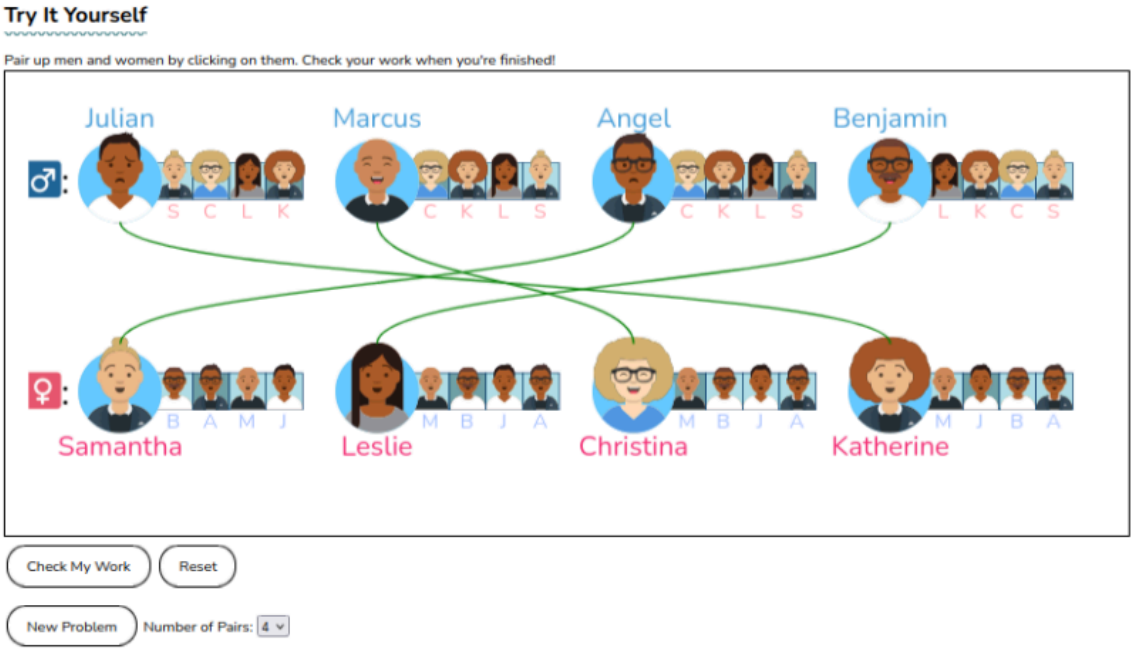
The Gale-Shapely Algorithm is mathematically proven to end (it does not go into an endless loop) and it always finds a stable solution.

Further it can be mathematically proven that the group in the graph that is able to initiate (in the classic example that is the men) "Among all possible different stable matchings, it always yields the one that is best for all men among all stable matchings, and worst for all women."

executed with men initiating



executed with women initiating



but what if we're not all heterosexual??

example that shows the algorithm without gender/heterodivisions → at the blackboard

the Gale-Shapely Algorithm (the historical Introduction)

The Algorithm was Developed by the two American game theorists (a type of mathematician) **David Gale** and **Lloyd Shapely** while working for the **RAND corporation** in their journal article:

"College admissions and the stability of marriage"

in which they first outline the problem of how to match college applicants with Universities that they apply to.
They outline their algorithm that finds a result that matches every applicant with a university, but trying to optimize that applicants' preferences.

Secondly they apply this to the "marriage problem"

> "A certain community consists of n men and n women. Each person ranks those of the opposite sex in accordance with his or her preferences for a marriage partner. We seek a satisfactory way of marrying off all members of the community. ... we call a set of marriages *unstable* (and here the suitability of the term is quite clear) if under it there are a man and a woman who are not married to each other but prefer each other to their actual partners "

RAND Corporation

"RAND was created after individuals in the **War Department** the **Office of Scientific Research and Development**, and industry began to discuss the need for a private organisation to connect operational research with research and development decisions. The immediate impetus for the creation of RAND was that ongoing demobilisation meant the federal government was about to lose direct control of the vast amount of American scientific brainpower assembled to fight World War II.

RAND contributed to the doctrine of nuclear deterrence by mutually assured destruction based upon their work with game theory.

real world applications

- used matching medical graduates and hospitals, where they do their first residency
- college applicants and universities
- assigning users to servers in a large distributed Internet service. Content delivery networks that distribute much of the world's content and services solve this problem between users and servers to enable users to be matched up with servers that can provide the requested web pages, videos, or other services with minimum latency
- allegedly used in the Dating App Hinge

gap & reflections

What do you think of the term "stability"? Does it sound like a desireable goal for a relationship?

What can we learn from an algorithm that is used for markets, being used in Dating (Apps)?

Extra Credit: RAND Corp

RAND was not the first think tank, but during the 1960s, it was the first to be regularly referred to as a "think tank".1(https://en.wikipedia.org/wiki/RAND_Corporation#cite_note-Medvetz-1) Accordingly, RAND served as the "prototype" for the modern definition of that term.

Chief strategist Herman Kahn also posited the idea of a "winnable" nuclear exchange in his 1960 book *On Thermonuclear War*. This led to Kahn's being one of the models for the titular character of the film *Dr. Strangelove*, in which RAND is spoofed as the "BLAND Corporation"

Emerging Technologies and Innovation

RAND has examined the implications of emerging technologies such as artificial intelligence, cybersecurity threats, and autonomous systems. It was accused of working too closely with Open Philanthropy in its work on AI, at the risk of losing its independence.53(https://en.wikipedia.org/wiki/RAND_Corporation#cite_note-53)[[54]](https://en.wikipedia.org/wiki/RAND_Corporation#cite_note-54)[[55]](https://en.wikipedia.org/wiki/RAND_Corporation#cite_note-55) RAND employees have expressed concerns to Politico about the organization's objectivity after it was revealed that RAND helped draft the Executive Order on AI, following over $15 million in funding from a Facebook founder-backed Open Philanthropy.56(https://en.wikipedia.org/wiki/RAND_Corporation#cite_note-56)

Notable participants

- Richard Bellman: Mathematician known for his work on dynamic programming
- Samuel Cohen: inventor of the neutron bomb in 1958
- George Dantzig: mathematician, creator of the simplex algorithm for linear programming
- Hubert Dreyfus: philosopher and critic of artificial intelligence
- Daniel Ellsberg: economist and leaker of the Pentagon Papers
- Francis Fukuyama: academic and author of *The End of History and the Last Man*
- Margaret Mead: U.S. anthropologist
- John Forbes Nash, Jr.: mathematician, won the Nobel Prize in Economics
- John von Neumann: mathematician, pioneer of the modern digital computer
- Lloyd Shapley: mathematician and game theorist, won the Nobel Prize in Economics